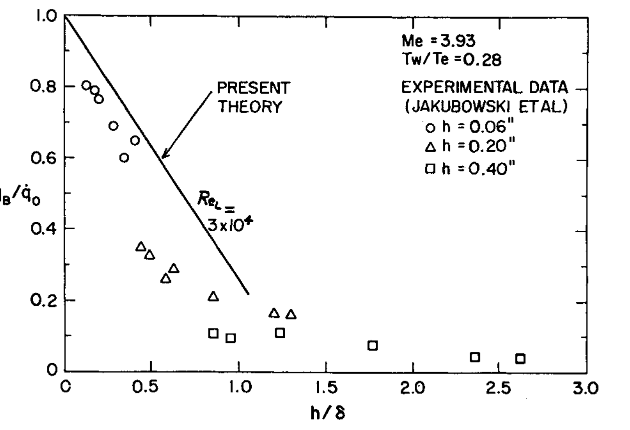

Fig. 5 Base heat transfer vs step height/boundary-layer thickness ratio.

The corresponding small-scale wall pressure solution has the form

$$p_w'/\gamma P_o \approx -(4h/\pi y_s)M_o^2(y^*)e^{kx/y_s} \tag{2}$$

in terms of the boundary-layer sonic height y_s where $k = 1$ and -3 for $x < 0$ and $x > 0$, respectively. This variation is continuous past the step (albeit with a discontinuous axial *gradient*) and acts to locally smooth out the large scale pressure jump illustrated in Fig. 1. Moreover, whereas the lateral pressure gradient in the large scale is negligibly small, it is quite large and rapidly varying (in fact comparable to the local axial pressure gradient) near the corner, as shown in Fig. 3.

Assuming laminar boundary-layer flow, the following useful closed-form approximation for the base pressure $p_B = p_w'(x \to 0^+)$ can be derived[2] from Eq. (2):

$$\frac{p_B}{p_o} \approx 1 - C_1 \frac{M_e^{\,3}\, Re_L^{\,1/4}}{(M_e^2-1)^{1/4}}\frac{h}{L} = 1 - C_1\Lambda \tag{3}$$

where the constant C_1 depends on T_w/T_e. Equation (3) indicates that the quantity Λ is a basic correlating parameter for the base pressure. This is verified in Fig. 4, where several sets of experimental data[4,5] are shown to correlate in terms of a single curve function of Λ over a moderately wide range of laminar flow conditions.

Solution of the energy equation in the heat-conducting disturbance sublayer near the wall gives the corresponding heat transfer behavior.[2] In the small-scale approximation near the step, this yields the following approximate laminar base heat-transfer expression:

$$\frac{q_B}{q_{w_o}} = 1 - C_2 \frac{M_e^{\,2}\, Re_L^{\,5/8}}{(M_e^{\,2}-1)^{1/2}}\frac{h}{L} \tag{4}$$

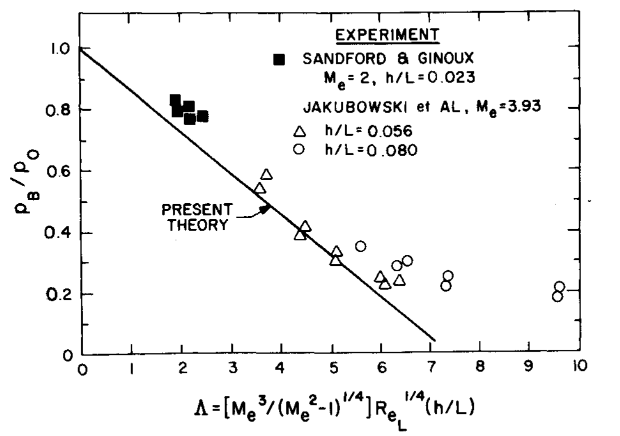

Fig. 4 Base pressure correlation for laminar supersonic flow.

which, as expected, exhibits a much stronger dependence on the Reynolds number than base pressure. This theoretical result is plotted vs h/δ in Fig. 5 along with some recent experimental measurements.[4] Although the predicted linear behavior at small h/δ is confirmed, the theory overestimates the heat transfer noticeably more than it does base pressure. This is to be expected since the presence of a recirculation zone downstream of the step (which is not accounted for in the theory) significantly reduces base heating for the cited step heights.

References

[1] Inger, G. R., "Three-Dimensional Heat Transfer and Ablation Disturbances in High Speed Flows," *AIAA Journal*, Vol. 10, No. 12, Dec. 1972, pp. 1641–1646.

[2] Inger, G. R., "Theory of Supersonic Laminar Non-Adiabatic Boundary Layer Flow Past Small Rearward-Facing Steps Including Suction," Rept. VPI-E-72-17, Aug. 1972, Virginia Polytechnic Institute and State University, Blacksburg, Va.

[3] Wu, J. M., Su, M., and Moulden, T. H., "On the Near Flow Field Generated by the Supersonic Flow over Rearward-Facing Steps," ARL-71-0243, Nov. 1971, Aerospace Research Labs., Wright-Patterson Air Force Base, Ohio.

[4] Jakubowski, A. K., Brown, R. D., Kirchner, R. D., and Lewis, C. H., "Experimental Investigation of Heat Transfer and Pressure Distributions in Laminar Separated Flows Downstream of a Rearward-Facing Step," Aerospace Engineering Report, July 1973, Virginia Polytechnic Institute and State University, Blacksburg, Va.

[5] Sandford, J. and Ginoux, J. J., "Laminar, Transitional, and Turbulent Heat Transfer Behind a Backward-Facing Step in Supersonic Flow," TN 38, Oct. 1968, von Kármán Institute, Brussels, Belgium.

Generalized Sturm-Liouville Procedure for Composite Domain Anisotropic Transient Conduction Problems

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Introduction

THE transient and steady-state temperature distribution in composite configurations consisting of several distinct thermally anisotropic subdomains have numerous applications to heat-transfer problems in re-entry vehicles, air frames, nuclear reactors, and the like. Apart from the purely numerical approaches such as the finite element[1–3] and difference procedures,[4] several purely analytical techniques[4–8] are also available. A comprehensive survey of the above noted analytical procedures has been reported by Ozisik.[4] With few exceptions,[9,10] most applications of analytical techniques have been limited to isotropic 1-D laminated domain problems. This is partly due to the added analytical complexity caused by anisotropy as well as the general lack of attention given to such thermal material properties. Due to the increased usage of inherently thermally anisotropic materials in a variety of applications, this situation is changing.

In this context, the purpose of the present Note will be to extend the relatively simple and straightforward Vodicka-Tittle[4–6] orthogonal expansion technique to 3-D configurations consisting of finitely many distinct fully anisotropic subdomains. As will be seen in the development, the procedure developed herein is based on a 3-D piecewise weighted orthogonality

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principle which in the 1-D isotropic case reduces to the Vodicka-Tittle relation.[4-6]

Development

Consider a 3-D composite configuration consisting of L distinct subdomains whose volume R is given by

$$R = \sum_{l=1}^{L} R_l \tag{1}$$

such that R_l denotes the volume of the lth distinct subdomain whose surface area ∂R_l has the form

$$\partial R_l = \sum_{k=e,f,g\ldots} \partial R_{lk} \tag{2}$$

where ∂R_{lk} denotes that portion of ∂R_l shared in common with ∂R_k. To simplify the ensuing development, with no loss in generality, the governing conduction equation for the lth subdomain is given in its Cartesian form, hence,

$$(\kappa_{mn}{}^{(l)} T_{,n}{}^{(l)})_{,m} + Q^{(l)} = (\rho c)^{(l)} T_{,t}{}^{(l)} \tag{3}$$

where $\kappa_{mn}{}^{(l)}$, $T^{(l)}$, $Q^{(l)}$, $t, \rho^{(l)}$, and $c^{(l)}$, respectively, represent the conductivity tensor, temperature, heat generation, time, density, and specific heat. As Cartesian tensor notation is being used herein, $(\ )_{,i}$ denotes partial differentiation with respect to the spacial variables x_m; $m = 1,2,3$ and $(\ )_{,t}$ represents time differentiation. The interdomain conditions associated with Eq. (3) on ∂R_{lk} take the form

$$\eta_m{}^{(l)} \kappa_{mn}{}^{(l)} T_{,n}{}^{(l)} = \eta_m{}^{(k)} \kappa_{mn}{}^{(k)} T_{,n}{}^{(k)} \tag{4}$$

$$T^{(l)} = T^{(k)} \tag{5}$$

where $\eta_m{}^{(l)}$ denotes the outward facing unit normal to ∂R_l. For subdomains with external exposure, $\partial R_{l\,\mathrm{ext}}$, the boundary conditions associated with Eq. (3) are given by

$$\eta_m{}^{(l)} \kappa_{mn}{}^{(l)} T_{,n}{}^{(l)} + H^{(l)} T^{(l)} = g^{(l)} \tag{6}$$

such that $H^{(l)}$ can be so chosen as to yield either Dirichlet, Neumann, or Cauchy type boundary conditions.

To initiate the development, $T^{(l)}(x_1, x_2, x_3, t)$ is separated into space and time variables and represented in an infinite series in the form

$$T^{(l)} = \sum_{p=1}^{\infty} \tau_p{}^{(l)}(x_1, x_2, x_3)\Gamma_p{}^{(l)}(t) \tag{7}$$

In terms of Eqs. (5) and (7) it immediately follows that on ∂R_{lk}

$$\sum_{p=1}^{\infty} \tau_p{}^{(l)}\Big|_{\partial R_{lk}} \Gamma_p{}^{(l)}(t) = \sum_{p=1}^{\infty} \tau_p{}^{(k)}\Big|_{\partial R_{lk}} \Gamma_p{}^{(k)}(t) \tag{8}$$

or following the usual Fourier procedure

$$\tau_p{}^{(l)}|_{\partial R_{lk}} \Gamma_p{}^{(l)}(t) = \tau_p{}^{(k)}|_{\partial R_{lk}} \Gamma_p{}^{(k)}(t) \tag{9}$$

Since t is free to range $t \in [0, \infty)$, Eq. (9) remains an identity iff

$$\Gamma_p{}^{(l)}(t) = \Gamma_p{}^{(k)}(t) = \Gamma_p(t); \quad l = 1, 2 \ldots, L \tag{10}$$

For the homogeneous case, $\Gamma_p(t) \propto \exp(\lambda_p{}^{(l)} t)$, thus in terms of Eq. (10) it follows that

$$\lambda_p{}^{(l)} = \lambda_p{}^{(k)} = \lambda_p; \quad l = 1, 2, \ldots, L \tag{11}$$

Hence Eq. (7) reduces to

$$T^{(l)} = \sum_{p=1}^{\infty} \tau_p{}^{(l)}(x_1, x_2, x_3)\Gamma_p(t) \tag{12}$$

Using Eq. (12) along with the homogeneous form of Eqs. (3–6), the following set of reduced equations can be obtained:

in R_l:
$$(\kappa_{mn}{}^{(l)} \tau_{,n}{}^{(l)})_{,m} = \lambda (\rho c)^{(l)} \tau^{(l)} \tag{13}$$

on ∂R_{lk}:
$$\eta_m{}^{(l)} \kappa_{mn}{}^{(l)} \tau_{,n}{}^{(l)} = \eta_m{}^{(k)} \kappa_{mn}{}^{(k)} \tau_{,n}{}^{(k)} \tag{14}$$

$$\tau^{(l)} = \tau^{(k)} \tag{15}$$

on $\partial R_{l\,\mathrm{ext}}$:
$$\eta_m{}^{(l)} \kappa_{mn}{}^{(l)} \tau_{,n}{}^{(l)} + H^{(l)} \tau^{(l)} = 0 \tag{16}$$

To determine the requisite orthogonality relation, it is assumed that for distinct eigenvalues λ_p and λ_q

$$\begin{aligned} (\kappa_{mn}{}^{(l)} \tau_{p,n}{}^{(l)})_{,m} &= \lambda_p (\rho c)^{(l)} \tau_p{}^{(l)} \\ (\kappa_{mn}{}^{(l)} \tau_{q,n}{}^{(l)})_{,m} &= \lambda_q (\rho c)^{(l)} \tau_q{}^{(l)} \end{aligned} \tag{17}$$

or after several manipulations, due to the symmetric nature of $\kappa_{ij}{}^{(l)}$

$$(\lambda_p - \lambda_q)(\rho c)^{(l)} \tau_p{}^{(l)} \tau_q{}^{(l)} = [\kappa_{mn}{}^{(l)}(\tau_{p,n}{}^{(l)} \tau_q{}^{(l)} - \tau_{q,n}{}^{(l)} \tau_p{}^{(l)})]_{,m} \tag{18}$$

Assuming that R_l is a closed bounded region whose boundary is a piecewise smooth orientable surface, the integration of Eq. (18) over R_l yields

$$(\lambda_p - \lambda_q) \int_{R_l} (\rho c)^{(l)} \tau_p{}^{(l)} \tau_q{}^{(l)} dV = \sum_{k=e,f,g,\ldots} \int_{\partial R_{lk}} \kappa_{mn}{}^{(l)} (\tau_{p,n}{}^{(l)} \tau_q{}^{(l)} - \tau_{q,n}{}^{(l)} \tau_p{}^{(l)}) \eta_m{}^{(l)} dS \tag{19}$$

From the nonzero nature of the right-hand side of Eq. (19), it follows that for the multisubdomain case ($L > 1$), Eq. (13) is nonself-adjoint in the traditional sense. Summing Eq. (19) over all l yields

$$(\lambda_p - \lambda_q) \sum_{l=1}^{L} \int_{R_l} (\rho c)^{(l)} \tau_p{}^{(l)} \tau_q{}^{(l)} dV = \sum_{l=1}^{L} \sum_{k=e,f,g,\ldots} \int_{\partial R_{lk}} \kappa_{mn}{}^{(l)} (\tau_{p,n}{}^{(l)} \tau_q{}^{(l)} - \tau_{q,n}{}^{(l)} \tau_p{}^{(l)}) \eta_m{}^{(l)} dS \tag{20}$$

To reduce the right-hand side of Eq. (20), in terms of Eqs. (14) and (15), the following identity can be established

$$\eta_m{}^{(l)} \kappa_{mn}{}^{(l)} (\tau_{p,n}{}^{(l)} \tau_q{}^{(l)} - \tau_{q,n}{}^{(l)} \tau_p{}^{(l)}) = \eta_m{}^{(k)} \kappa_{mn}{}^{(k)} (\tau_{p,n}{}^{(k)} \tau_q{}^{(k)} - \tau_{q,n}{}^{(k)} \tau_p{}^{(k)}) \tag{21}$$

Using the preceding identity together with the external conditions [Eq. (16)], Eq. (20) reduces to

$$\sum_{l=1}^{L} \int_{R_l} (\rho c)^{(l)} \tau_p{}^{(l)} \tau_q{}^{(l)} dV = \begin{cases} o, p \neq q \\ \neq o, p = q \end{cases} \tag{22}$$

Interestingly, as can be seen from the abovementioned development, the piecewise-weighted orthogonality relation represented by Eq. (22) applies to fully anisotropic as well as to monoclinic, orthotropic, and isotropic media. Hence although Eq. (13) is nonself-adjoint in the local sense, they are self-adjoint in the global sense described by Eq. (22).

Before formally establishing the generalized Sturm-Liouville procedure, to limit the search for eigenvalues arising out of the eigenvalue problem depicted by Eqs. (13–16), the realness and negative definiteness of λ will be established. To ascertain the realness of λ assume that

$$\lambda = Re\{\lambda\} + jIm\{\lambda\} \tag{23}$$

$$\tau^{(l)} = Re\{\tau^{(l)}\} + jIm\{\tau^{(l)}\} \tag{24}$$

such that $j = (-1)^{1/2}$. Using Eqs. (23) and (24), Eqs. (13–16) reduce to

in R_l:

$$\begin{aligned} (\kappa_{mn}{}^{(l)} Re\{\tau_{,n}{}^{(l)}\})_{,m} &= (\rho c)^{(l)}(Re\{\lambda\} Re\{\tau^{(l)}\} - Im\{\lambda\} Im\{\tau^{(l)}\}) \\ (\kappa_{mn}{}^{(l)} Im\{\tau_{,n}{}^{(l)}\})_{,m} &= (\rho c)^{(l)}(Re\{\lambda\} Im\{\tau^{(l)}\} + Im\{\lambda\} Re\{\tau^{(l)}\}) \end{aligned} \tag{25}$$

on ∂R_{lk}:

$$\begin{aligned} \eta_m{}^{(l)} \kappa_{mn}{}^{(l)} Re\{\tau_{,n}{}^{(l)}\} &= \eta_m{}^{(k)} \kappa_{mn}{}^{(k)} Re\{\tau_{,n}{}^{(k)}\} \\ \eta_m{}^{(l)} \kappa_{mn}{}^{(l)} Im\{\tau_{,n}{}^{(l)}\} &= \eta_m{}^{(k)} \kappa_{mn}{}^{(k)} Im\{\tau_{,n}{}^{(k)}\} \end{aligned} \tag{26}$$

$$Re\{\tau^{(l)}\} = Re\{\tau^{(k)}\}, \quad Im\{\tau^{(l)}\} = Im\{\tau^{(k)}\} \tag{27}$$

on $\partial R_{l\,\mathrm{ext}}$:

$$\begin{aligned} \eta_m{}^{(l)} \kappa_{mn}{}^{(l)} Re\{\tau_{,n}{}^{(l)}\} + H^{(l)} Re\{\tau^{(l)}\} &= 0 \\ \eta_m{}^{(l)} \kappa_{mn}{}^{(l)} Im\{\tau_{,n}{}^{(l)}\} + H^{(l)} Im\{\tau^{(l)}\} &= 0 \end{aligned} \tag{28}$$

Performing several manipulations on Eqs. (25) it follows that

$$(\rho c)^{(l)} Im\{\lambda\}(Re^2\{\tau^{(l)}\} + Im^2\{\tau^{(l)}\}) = [\kappa_{mn}{}^{(l)}(Im\{\tau_{,n}{}^{(l)}\} Re\{\tau^{(l)}\} - Re\{\tau_{,n}{}^{(l)}\} Im\{\tau^{(l)}\})]_{,m} \tag{29}$$

or upon integration over R_l and summation over all l

$$Im\{\lambda\} \sum_{l=1}^{L} \int_{R_l} (\rho c)^{(l)}(Re^2\{\tau^{(l)}\} + Im^2\{\tau^{(l)}\}) dV = \sum_{l=1}^{L} \sum_{k=e,f,g,\ldots} \int_{\partial R_{lk}} \kappa_{mn}{}^{(l)}(Im\{\tau_{,n}{}^{(l)}\} Re\{\tau^{(l)}\} - Re\{\tau_{,n}{}^{(l)}\} Im\{\tau^{(l)}\}) \eta_m{}^{(l)} dS \tag{30}$$

In the spirit of the development of Eq. (21), using Eqs. (26) and (27) the following identities can be established on ∂R_{lk}

$$\eta_m^{(l)}\kappa_{mn}^{(l)}(Im\{\tau_{,n}^{(l)}\}Re\{\tau^{(l)}\} - Re\{\tau_{,n}^{(l)}\}Im\{\tau^{(l)}\}) = \eta_m^{(k)}\kappa_{mn}^{(k)}(Im\{\tau_{,n}^{(k)}\}Re\{\tau^{(k)}\} - Re\{\tau_{,n}^{(k)}\}Im\{\tau^{(k)}\}) \quad (31)$$

Based on Eqs. (28) and (31), Eq. (30) reduces to the form

$$Im\{\lambda\}\sum_{l=1}^{L}\int_{R_l}(\rho c)^{(l)}(Re^2\{\tau^{(l)}\} + Im^2\{\tau^{(l)}\})dV \equiv 0 \quad (32)$$

Now since $(\rho c)^{(l)}$ and the integrals appearing in Eq. (32) are all positive definite, it follows that $Im\{\lambda\} \equiv 0$, hence λ is purely real.

Assuming that Eqs. (13–16) represent Euler equations, the following equivalent variational formulation can be written

$$\lambda = -\sum_{l=1}^{L}\int_{R_l}\kappa_{mn}^{(l)}\tau_{,m}^{(l)}\tau_{,n}^{(l)}dV \Big/ \sum_{l=1}^{L}\int_{R_l}(\rho c)^{(l)}(\tau^{(l)})^2 dV \quad (33)$$

Since both the numerator and denominator appearing in Eq. (33) are positive definite, it follows that all λ are negative definite.

In terms of orthogonality and variational relations given by Eqs. (22) and (33), standard procedures[11] can be used to verify the completeness of the $\tau_p^{(l)}$ eigenfunction set together with convergence characteristics of expansions in terms of these functions. Tacitly assuming that solutions for the eigenvalue problem denoted by Eqs. (13–16) exist, the solution to Eqs. (3–6) can be developed following the classical Sturm-Liouville procedure. Hence, considering that the external conditions, Eqs. (4–6), can be homogenized using standard procedures, Eqs. (3–6) reduce to

in R_l:
$$(\kappa_{mn}^{(l)}\tilde{T}_{,n}^{(l)})_{,m} + \tilde{Q}^{(l)} = (\rho c)^{(l)}\tilde{T}_{,t}^{(l)} \quad (34)$$

on ∂R_{lk}:
$$\eta_m^{(l)}\kappa_{mn}^{(l)}\tilde{T}_{,n}^{(l)} = \eta_m^{(k)}\kappa_{mn}^{(k)}\tilde{T}_{,n}^{(k)} \quad (35)$$
$$\tilde{T}^{(l)} = \tilde{T}^{(k)} \quad (36)$$

on $\partial R_{l\,\mathrm{ext}}$:
$$\eta_m^{(l)}\kappa_{mn}^{(l)}\tilde{T}_{,n}^{(l)} + H^{(l)}\tilde{T}^{(l)} = 0 \quad (37)$$

where $\tilde{T}^{(l)}$ is the reduced temperature and $\tilde{Q}^{(l)}$ is the modified heat generation. To solve Eqs. (34–37), $\tilde{Q}^{(l)}$ and $\tilde{T}^{(l)}(x_1, x_2, x_3, 0)$ are expanded as follows

$$\frac{1}{(\rho c)^{(l)}}\tilde{Q}^{(l)} = \sum_{p=1}^{\infty} q_p(t)\tau_p^{(l)} \quad (38)$$

$$\tilde{T}^{(l)}(x_1, x_2, x_3, 0) = \sum_{p=1}^{\infty} f_p\tau_p^{(l)} \quad (39)$$

where

$$\langle q_p, f_p\rangle = \frac{\sum_{l=1}^{L}\int_{R_l}(\rho c)^{(l)} \langle Q^{(l)}/(\rho c)^{(l)}, \tilde{T}(o)^{(l)}\rangle \tau_p^{(l)}dV}{\sum_{l=1}^{L}\int_{R_l}(\rho c)^{(l)}\tau_p^{(l)}\tau_p^{(l)}dV} \quad (40)$$

In terms of Eqs. (12, 38, and 39), Eqs. (34–37) reduce to

$$\Gamma_{p,t} - \lambda_p\Gamma_p - q_p = 0 \quad (41)$$

Solving Eq. (41) in terms of the initial condition denoted by Eq. (39) yields

$$\Gamma_p = f_p e^{\lambda_p t} + q_p * e^{\lambda_p t} \quad (42)$$

where * represents the Faltung. Hence, in terms of Eq. (42) the final solution of Eqs. (34–37) takes the form

$$T^{(l)} = \sum_{p=1}^{\infty}\tau_p^{(l)}(f_p e^{\lambda_p t} + q_p * e^{\lambda_p t}) \quad (43)$$

Because of the generality of the abovementioned development, for the 1-D case, the results derived herein reduce to the Vodicka-Tittle procedure. In particular, the 3-D piecewise weighted orthogonality relation, Eq. (22), reduces to its 1-D Vodicka-Tittle equivalent. One very important feature of the development given herein is that the eigenvalue problem represented by Eqs. (13–16) can be solved either analytically or numerically. Hence, the powerful finite element procedure can be used to obtain the eigenvalues and eigenfunctions of Eqs. (13–16). Using these together with the expansion denoted by Eq. (43), the solution of Eqs. (3–6) can be developed. In such a case, the number of terms retained in the series, Eq. (43), will depend on the fineness of the element breakdown.

References

[1] Zienkiewicz, O. C., *The Finite Element Method in Engineering Science*, McGraw-Hill, New York, 1971.

[2] Padovan, J., "Analysis of Heat Conduction in Anisotropic Axisymmetric Solids," presented at the 9th Annual Southeastern Seminar on Thermal Sciences, July 26–27, 1973, Old Dominion Univ., Norfolk, Va.

[3] Padovan, J., "Quasi-Analytical Finite Element Procedure for Conduction in Anisotropic Axisymmetric Solids," *International Journal for Numerical Methods in Engineering*, Vol. 8, 1974, pp. 297–312.

[4] Ozisik, M. N., *Boundary Value Problems of Heat Conduction*, International Textbook, Scranton, Pa., 1968.

[5] Vodicka, V., "Warmeleitung in Geschichten Kugel- und Zylinder Korpern," *Schweizer Archv.*, Vol. 10, 1950.

[6] Tittle, C. W., "Boundary Value Problems in Composite Media," *Journal of Applied Physics*, Vol. 36, 1965, pp. 1486–1488.

[7] Carslaw, H. S. and Jaeger, J. C., *Conduction of Heat in Solids*, Oxford University Press, London, 1959.

[8] Goodman, T. R., "The Adjoint Heat-Conduction Problems for Solids," ASTIA-AD254-769, (AFOSR-520), April 1961, Air Force Office of Scientific Research, Wright-Patterson Air Force Base, Ohio.

[9] Padovan, J., "Temperature Distributions in Anisotropic Shells of Revolution," *AIAA Journal*, Vol. 10, No. 1, Jan. 1972, pp. 60–64.

[10] Padovan, J., "Conduction in Anisotropic Composite Slabs and Cylinders," to be presented at the 5th International Conference of Heat Transfer, Sept. 1974, Tokyo, Japan.

[11] Courant, R. and Hilbert, D., *Methods of Mathematical Physics*, Interscience Publishers, New York, 1953.

Raman Scattering Applied to Hypersonic Air Flow

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Introduction

AN intimate knowledge of three-dimensional flowfields is necessary to determine the design and performance of future flight vehicles. Standard probing techniques which have been adequate to define simple flowfields are no longer useful when severe streamline curvature exists since the presence of the probe may alter the flowfield and the flow direction is not known a priori. The most desirable probe for three-dimensional flows would be nonintrusive and independent of flow direction; obviously, this describes an optical probing technique. In the present investigation, the Raman scattering technique was used to measure the local static temperature and gas number density over a flat plate in a Mach 5 nozzle of the Langley nozzle test chamber with air as the test gas. While this flowfield is not three-dimensional, the accuracy of the resulting measurements of density and temperature confirm the accuracy of the technique for more complicated flows (as long as the spatial resolution of the sample volume is sufficiently small).

Measurements were made in the inviscid flowfield of a sharp leading-edge flat-plate model at several angles of attack ($-5°$ to $15°$) and over a wide range of tunnel conditions (stagnation pressures P_o from 1.7×10^5 to 2.8×10^6 N/m^2 and stagnation temperatures T_o from 317° to 442°K). The measured values of

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